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ORIGINAL ARTICLE

Piecewise Fractional Analysis of Omicron Type Covid-19 Infection

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ABSTRACT

In this article, we investigate the new type of COVID-19 caused by the Omicron virus in the sense of piece-wise fractional derivative. The total interval is divided into two sub-intervals under fractional Caputo and Atangana Baleanu operators respectively. The whole model is divided into six compartments in which the agent of infection from Omicron is included. The proposed piecewise fractional model is tested for fixed points using the different theorems of fixed point theory. The approximate solution is carried out by the technique of the piecewise fractional Adams-Bashforth method. All the agents of the considered problem are tested for graphical representation. The first sub-interval is graphed for the Caputo derivative while the second interval dynamics are checked by Atangana Baleanu fractional operator. The numerical simulation results are established at different fractional orders along with the comparison of integer orders. This consideration will also show the cross-over behavior of the Omicron dynamics in human life and will be essential for its controlling and future prediction on various sub-intervals. The sensitivity of different parameters is also checked graphically.

Keywords: Piecewise Fractional Derivative, Fractional mathematical model Omicron; Qualitative analysis; Caputo derivative; Fractional Adams-Bashforth technique.

INTRODUCTION

Since the discovery of infection of COVID, various types of it, due to different viruses have also been tested like MERSE-COVID, COVID-19, Omicron type COVID-19, etc. The Omicron is a novel virus infection of SARS-CoV-2 caused by the virus known as the Omicron virus. This virus is related to the chain of SARS-CoV-2 (COVID-19), tested in the month of November 2021, in the country of South Africa. After the discovery of the said virus and quick expansion, its reached other continents of the world. The cases of the Omicron infection increase day by day. The infection of this virus is not more severe than the usual COVID-19 and its other types but the infection is very fast as compared with the other COVID-19 types. According to WHO it can affect the vaccinated persons and having not any proper signs of infection [1]. Still, some usual signs are tested in this infection like coughing, congestion, watery nose, body aches, etc. Like the bans on COVID-19, this infection also faces bans on overcrowding, unmasking, traffic bans and flight suspensions.

The diseases of COVID-19 have been analyzed by several scholars and biologists to control the infection and its further expansion in society. They tried and tried to develop the treatment and cure in the form of vaccines to vaccinate many peoples in order to minimize the number of infectious people and their future control. But still, with the duration of time and the emergency of the novel viruses of COVID-19, the globe is facing pandemics in many countries and societies. Mathematical models touched each infection in mathematical terms. Therefore, Some mathematical models in a natural order and non-natural orders are considered to investigate the COVID-19 pandemics. For example, the first infection of COVID-19 in Wuhan city of China in the sense of a very significant mathematical model is studied in [2]. The optimal control techniques for the eradication or controlling of the infection in Pakistan, by proposing the real COVID-19 classes have been discussed in [3]. The COVID-19 disease is converted to another healthy person very quickly, so the better and most effective framework is to minimize the infection, which is the self-isolation and quarantined technique, which is analyzed by a mathematical model established by scholars in [4]. The lock-down and its impacts on infection control have been investigated through a mathematical formulation technique in [5]. The researchers constructed an SEIR model using the realist data approach from France and Italy and established the disease control techniques [6]. Various reports related to COVID-19 cases and their formulation in Nigeria, with comparison, have been given in [7]. A global analysis on COVID- 19 to study the self-isolation, quarantined, and environmental vital wights have been pointed out in [8]. A Comprehensive discussion on COVID-19 in the framework of the fractional environment is carried out in [9]. The discussion of the COVID-19 disease modeling the realistic cases in Saudi Arabia has been established in [10].

NUMERICAL SIMULATION AND DISCUSSION

This section is devoted to piecewise derivative numerical simulation for the considered model of the new Omicron virus by partitioning the whole interval of [0, T] into two sub intervals [0, t_1], [t_1, T] respectively. We simulate our model for three different fractional orders and time intervals. We take data from [14] for different parameters and compartments of the proposed model as given in table 2.

Table 1: Initial and parameters numerical values for Omicron virus model [14].

Parameter	value	Parameter	value	Parameter	value
S_0	60069540	u	1 64.38×365	N	60140000
E_0	620000	α	0.7999	κ	0.7800;
Iα(0)	8000	τ	0.8999	Ψ	0.9566;
Is(0)	360	ϕ	0.0101	Δ_1	0.8447
IO(0)	100	Δ_2	0.0200	Δ_3	0.6746
R_0	0	δ_1	0.0015		

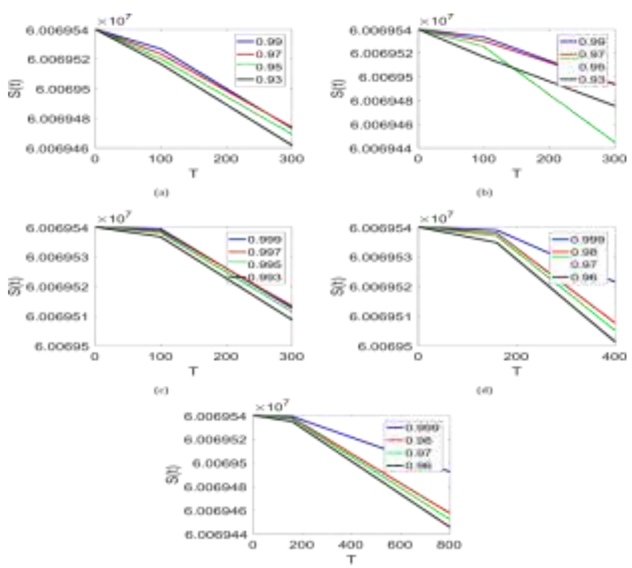

Figure 1: Dynamical behavior of susceptible individuals S(t) at different arbitrary fractional order r on sub interval [0, t_1] and [t_1, T] of [0, T], for different times durations

In figures 1a-1e, we draw the dynamics of the susceptible population on four different fractional orders and different time durations for two subintervals showing bending behavior. We also change the step size in the first two graphs showing that the curves are far away from each other in small step size and vice versa. This class shows less decay in the first interval while more in the second interval as its transfers to other compartments of the model for all different fractional orders and time duration. The decay is more at low fractional-order and less at high fractional orders.

In figures 2a-2e, we draw the dynamical representation of the Exposed population on four different fractional orders and different time durations for two subintervals showing bending behavior. We also change the step size in the first two graphs showing that the curves are far away from each other in small step size and vice versa. This class also decreases more in the first interval while small in the second interval as its transfers to another compartment of the model for all different fractional orders and time duration. The decay is more at low fractional-order and less at high fractional orders.

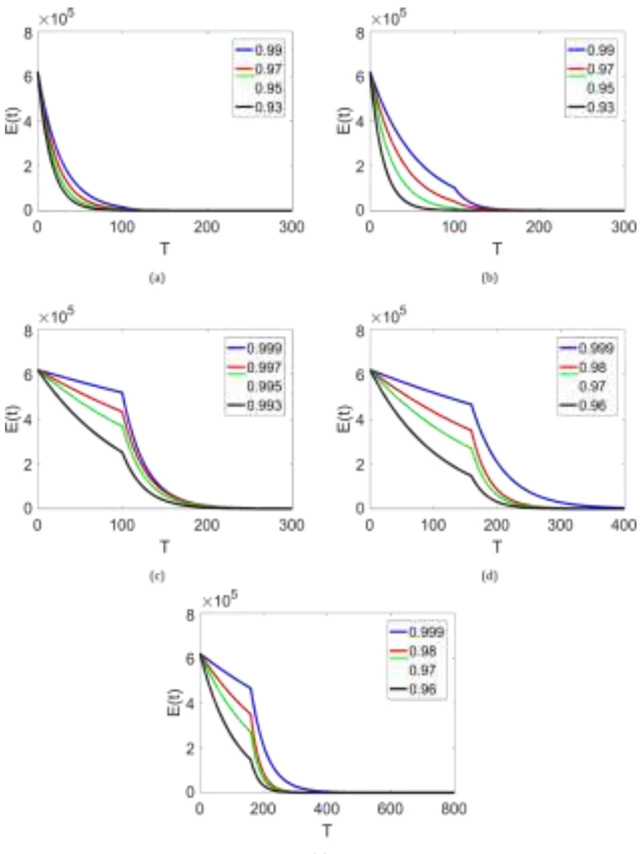

Figure 2: Dynamical behavior of Exposed individuals E(t) at different arbitrary fractional order r on sub interval [0, t_1] and [t_1, T] of [0, T], for different times durations

The figures 3a-3e show the dynamical behavior of Asymptomatic individuals on four different arbitrary orders and time durations for two subintervals showing bending behavior. We also variate the step size in the first two figures showing that the curves are far away from each other in small step size and vice versa. The population of the said class grows and after reaching the peak value it declines along with bending in the middle. The said class increases by transferring the population from the first two classes to it. The increase is more at high fractional-order and less at low fractional orders while the decrease is reverse of it.

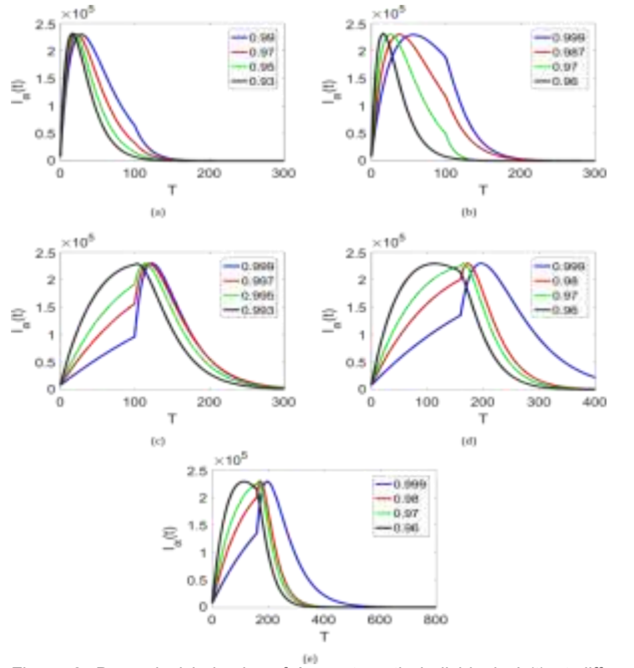

Figure 3: Dynamical behavior of Asymptomatic individuals $I_\alpha(t)$ at different arbitrary fractional order r on sub interval [0, t_1] and [t_1, T] of [0, T], for different times durations

The figures 4a-4e show the dynamics of Symptomatic individuals on four different arbitrary orders and time durations for two subintervals showing bending behavior. We variate the step size in the first two figures to know about the sensitivity of the model for the step size. The population of the said class grows and after reaching the maximum value it declines along with bending in the middle. The said class increases by transferring the population from the first two classes to it. The increase is more at high fractional-order and less at low fractional orders while the decrease is against it.

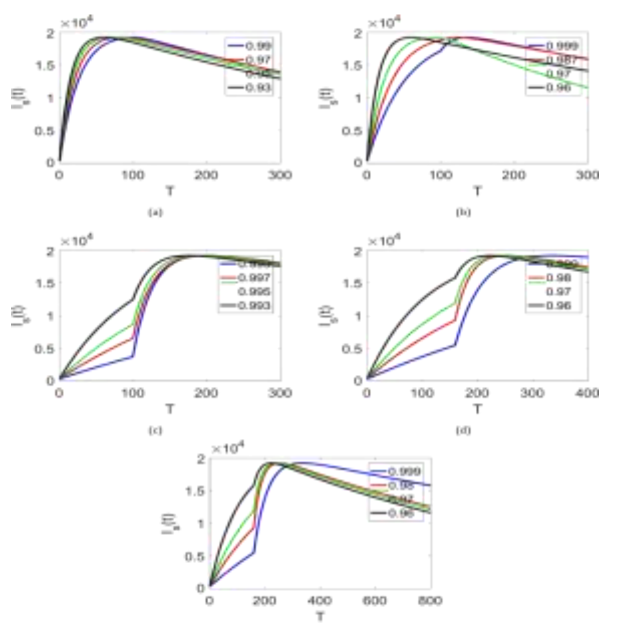

Figure 4: Dynamical behavior of Symptomatic individuals $I_s(t)$ at different arbitrary fractional order r on sub interval [0, t_1] and [t_1, T] of [0, T], for different times durations

The figures 5a-5e shows the dynamics of Omicron virus-infected individuals on four different arbitrary orders and time durations for two sub intervals showing bending behavior. We change the step size in the first two figures to know about the sensitivity of it. The population of the said class grows and after reaching the maximum value it declines along with bending or cross over properties at middle. The said class increases by transferring the population from first two classes to it in the form of infection cased by Omicron virus. The increase is more at high fractional-order and less at low fractional orders while the decrease is opposite of it. With passage of time the class vanishes or reduced to minimum level.

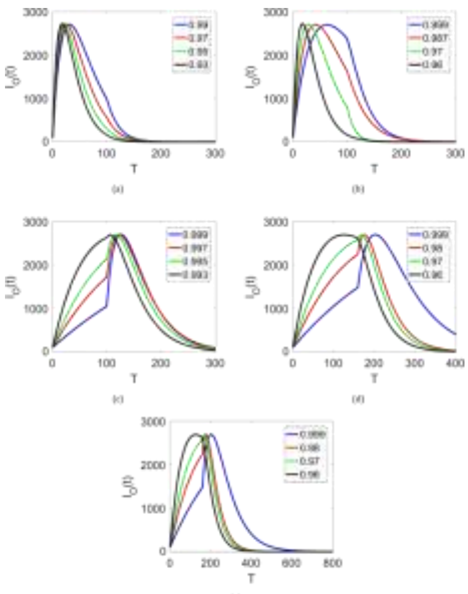

Figure 5: Dynamical be havior of Infection with Omicron Virus $I_O(t)$ at different arbitrary fractional order r on sub interval [0, t_1] and [t_1, T] of [0, T], for different times durations

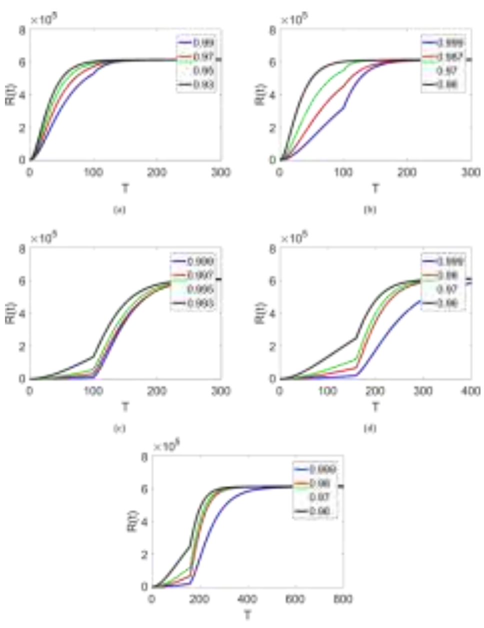

Figure 6: Dynamical behavior of Recovered individuals R(t) at different arbitrary fractional order r on sub interval [0, t_1] and [t_1, T] of [0, T], for different times durationsConclusion

The Figures 6a-6e represent the dynamics of Recovery from all types of infection given in the model on four different fractional orders and time durations for two subintervals showing some crossover dynamics. We also change the step size in the first two figures to know about its sensitivity of it. The said class grows and after reaching the maximum value it is stable with bending or cross-over properties in the middle. The said class increases by transferring the population from all infection classes to it in the form of recovery. The increase is more at high fractional-order and less at low fractional orders while the decrease is the opposite of it.

In the developed manuscript we have investigated a scheme of a new piece-wise non-integer order deriva- tive model of Omicron virus infection under Caputo and Atangana Balaneu Caputo fractional operators re- spectively. The dynamics for the said model have been carried out for two subintervals by portioning the whole interval to produce the piecewise crossover properties. In this article, we will be able to give predic- tions about infection dynamics at two different intervals with different behaviors. In the first interval, the decrease and increase in all compartments are different from the second one. The qualitative techniques of both the intervals for the considered model solution have been developed using the concept of fixed point theory. The numerical solution is evaluated for the model using Newton’s polynomial procedure for both sub-interval in Caputo and ABC framework of order r. The numerical simulation of all the six compartments has been drawn for five different data of fractional orders, step size and time duration. The crossover effects are shown by the termination of the first interval, describing the characteristics of the piecewise derivative behaviors. This type of analysis can be applied to real-world dynamic phenomena where abrupt or sudden variation occurs. This investigation is a more realistic approach as the dynamics are changing differently on different time durations. Such analysis describes the crossover properties which are still not given in deterministic and stochastic problems of both integer and fractional orders.

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